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Short communication

I. A simplified model for determining capacity usage and battery size for hybrid and plug-in hybrid electric vehicles

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ABSTRACT

We develop a simplified model to examine the effect of the shape and magnitude of the battery pulse-power capability on capacity usage and battery size. The simplified model expresses the capacity usage and a dimensionless battery area in terms of a dimensionless energy-to-power ratio and a parameter that characterizes the shape of the pulse-power capability. We also present dimensional results that show how the capacity usage depends on the equivalent-electric range and separator area, and how the battery area depends on the equivalent-electric range. Key results include the presence of a Langmuir-like relationship between the capacity usage and the dimensionless energy-to-power ratio, and a linear relationship between the dimensionless energy-to-power ratio and a dimensionless area, with a slope and offset that depend on the shape of the pulse-power capability. We also found that a flat pulse-power capability curve increases capacity usage and decreases battery size, and that two important parameters for battery design are $(U-V_{min})V_{min}/R$, which reflects the maximum power capability, and $Q\langle V\rangle$, which reflects the battery energy. The results and analysis contained herein are used to help interpret the results from a combined battery and vehicle model, presented in a companion paper.

1. Introduction

Increasing attention is being given to energy use in the transportation sector, and some of this attention is focused on the reductions in fuel consumption that can result from hybrid electric vehicles (HEVs) and plug-in hybrid electric vehicles (PHEVs). HEVs can vary according to the degree of hybridization, but offer improvements in efficiency of up to approximately 50% [1]. Another major justification for the development of PHEVs is the ability to accomplish fuel switching, for example replacing oil with electricity (derived from natural gas, coal, or renewables). An ultimate goal is to electrify the transportation industry and derive the electricity from renewable sources. This would simultaneously address the issues of “energy security” and climate change. Two of the main challenges that hinder the growth of the HEV and PHEV technologies are cost and the reduced cargo space that results from the inclusion of a large battery in a vehicle. It should also be noted that cost and size are linked; in general a larger battery costs more. Thus, there is significant pressure to use a battery that, while meeting performance requirements, is as small as possible. One way to achieve a smaller battery is to find a way to increase the capacity usage [2]. This could be achieved with the use of better control algorithms, or with a chemistry that provides a relatively flat pulse-power capability [3].

Here we present a simplified model for capacity usage and battery size in a HEV and PHEV. The goals are to examine the underlying relationships between key system variables in order to improve understanding, and assist with the interpretation of the results from a companion paper that treats a specific set of cell chemistries with a detailed battery model combined with a simple vehicle model [4]. This paper is divided into three sections. The first section discusses maximum power capability, the second section develops a simplified model for PHEVs and expresses results in terms of a dimensionless energy-to-power ratio, and the third section applies the simplified model to practical questions encountered by battery designers.

2. Defining the maximum power

Assuming a linear system (one for which the resistance is independent of applied current), the maximum power per unit area of a battery is given by

$$\frac{P_{max,discharge}}{A} = \frac{U^2}{4R}, \tag{1}$$

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Nomenclature

List of symbols

A	total separator area of a battery (m^2)
B	slope of the pulse-power capability curve
c_1	constant calculated from the position of V_{min}
c_2	constant calculated from the position of V_{max}
E	energy required at the wheels during a driving cycle (J)
E'	battery energy for a vehicle to travel a given distance ($J\,m^{-1}$)
i	current density ($A\,m^{-2}$)
L	equivalent-electric driving distance (m)
P_{max}	maximum power required from a battery for a given configuration (W)
$(P/A)^0$	maximum specific pulse-power capability for a given chemistry and cell design ($W\,m^{-2}$)
Q	specific coulombic capacity ($C\,m^{-2}$)
R	cell resistance (ohm m^2)
SOD	state-of-discharge
U	equilibrium potential (V)
V	instantaneous cell potential (V)
$\langle V \rangle$	average cell potential (V)
V_{max}	upper cutoff potential (V)
V_{min}	lower cutoff potential (V)

where R is the resistance and U is the equilibrium potential. The maximum power occurs at a current of $U/2R$, at which point the cell potential, V, is equal to $U/2$. There is an additional constraint on practical systems, namely the need to stay above the lower cutoff potential, V_{min}, and below the upper cutoff potential, V_{max}. If the maximum discharge power given by Eq. (1) occurs at a potential below the lower cutoff potential, then the maximum allowable power occurs at the lower cutoff potential. Note that because the value of the lower cutoff potential is often set to 55% of the upper voltage cutoff (for vehicle applications), and the value of U needs to lie between the upper and lower cutoff potentials, the actual maximum discharge power will generally have a value below the theoretical maximum power [5]. If the value of the upper and lower cutoff potentials are chosen so that U is halfway between, $V_{min} = 0.775U$.

In situations where the maximum power occurs at the cutoff potential, one of the following equations should be used [6]:

$$\frac{P_{max,discharge}}{A} = V_{min} i = V_{min}\left(\frac{U - V_{min}}{R}\right), \tag{2}$$

$$\frac{P_{max,charge}}{A} = V_{max} i = V_{max}\left(\frac{U - V_{max}}{R}\right). \tag{3}$$

$P_{max,charge}$ and $P_{max,discharge}$ are maximum required values of the charge and discharge power applied to a battery. The United States Council for Automotive Research (USCAR) has provided values of $P_{max,charge}$ and $P_{max,discharge}$ for a variety of vehicle configurations, such as HEVs and PHEVs [5,7]. Because the cutoff potentials can be expressed as a fraction of the equilibrium potential, U, we can rewrite Eqs. (2) and (3) as

$$\frac{P_{max,discharge}}{A} = \frac{c_1 U^2}{R}, \tag{4}$$

$$\frac{P_{max,charge}}{A} = \frac{c_2 U^2}{R}, \tag{5}$$

where c_1 and c_2 contain the location of V_{min} and V_{max} relative to U. Note that the maximum discharge power typically occurs near a state-of-discharge (SOD) of 0, while the maximum charge power occurs near a SOD of 1.0.

3. Simplified model for capacity usage in HEVs and PHEVs

First, define a linear pulse-power capability of a battery as

$$\frac{P_{max}/A}{(P/A)^0} = 1 - B\ \Delta\text{SOD}. \tag{6}$$

P_{max} is the maximum power required for a particular vehicle configuration, and is identical to $P_{max,discharge}$ or $P_{max,charge}$ described above, depending on whether the battery is being discharged or charged. A is the total separator area of the battery. $(P/A)^0$ is an area-specific maximum pulse-power capability and is a property of the unit cell, depending on the chemistry and cell design (*e.g.*, increasing the electrode thicknesses increases $(P/A)^0$). B is a parameter that characterizes the shape of the linear pulse-power capability; higher values of B correspond to more steeply sloped pulse-power capabilities. ΔSOD is the state-of-discharge range over which a given power requirement can be met. We stress that this linear and symmetric definition of the pulse-power capability given by Eq. (1) is not meant to reflect actual experimental systems; rather, we use a linear and symmetric definition because it simplifies the mathematics and allows us to easily develop the key relationships among the design variables and parameters of interest. We have plotted this equation in Fig. 1 for three arbitrarily selected values of B (B = 0.75, 1.0, and 1.25). Three lines start from the middle at $(P_{max}/A) = (P/A)^0$, and descend on either side with a slope of B or $-B$. These lines can be interpreted as giving the power capability of three hypothetical chemistries with different linear pulse-power capabilities (resulting, for example, from differently shaped equilibrium potential and resistance curves). Fig. 1 shows that as the total separator area, A, is increased, the state-of-discharge range over which the power requirement can be met, ΔSOD, increases.

Second, specify the equivalent-electric range, L. If E' is the battery energy required to go a unit distance ($J\,m^{-1}$), then the energy required from the battery is $E = E'L$ This can be approximated by

$$E = E'L = AQ\langle V \rangle\ \Delta\text{SOD}, \tag{7}$$

where Q is the area-specific capacity corresponding to ΔSOD = 1. $\langle V \rangle$ is the average cell potential during the sequence of charge and discharge pulses during a drive cycle, and is constrained between V_{max} and V_{min}. It can be approximated by U, assuming that the battery receives both charge and discharge pulses (alternatively, it can be

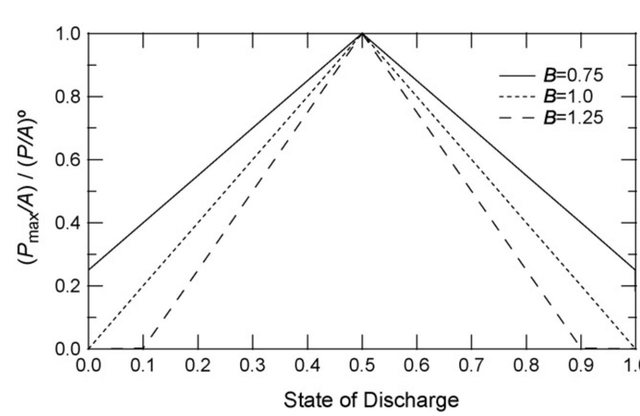

Fig. 1. Pulse-power capability of a generic cell. The maximum value of $(P_{max}/A)/(P/A)^0$ is equal to 1.0 at SOD = 0.5 because at that point ΔSOD = 0. Different values of B correspond to different shapes of the pulse-power capability. The pulse-power capability for SOD < 0.5 can be considered to be limited by charge, while for SOD > 0.5 it is limited by discharge.

approximated as V_{min}; the choice does not qualitatively affect the results). One can solve these equations for A and ΔSOD in terms of L. Starting with

$$A = \frac{P_{max}/(P/A)^0}{1 - B\ \Delta \mathrm{SOD}}, \tag{8}$$

substitute this into Eq. (7) and solve for ΔSOD:

$$E'L = Q\left\langle V\right\rangle \frac{P_{max}/(P/A)^0}{1 - B\,\Delta \mathrm{SOD}}\,\Delta \mathrm{SOD}, \tag{9}$$

or

$$\Delta \mathrm{SOD} = \frac{E'L}{E'LB + Q\left\langle V\right\rangle P_{max}/(P/A)^0}, \tag{10}$$

or

$$B\,\Delta \mathrm{SOD} = \frac{x}{1+x}, \tag{11}$$

where

$$x = \frac{E'LB(P/A)^0}{P_{max}Q\left\langle V\right\rangle}. \tag{12}$$

Eq. (11) looks like a Langmuir isotherm. x can be considered a dimensionless energy-to-power ratio, and for a given value of $P_{max}Q\langle V\rangle$ (that is, for a fixed power requirement and area-specific battery energy) can be considered as a dimensionless equivalent-electric driving range. Fig. 2 shows a plot of Eq. (11). The curve rises from 0 to 1 as x goes from 0 to infinity. However, for any curve with $B < 1$, say $B = 0.75$, draw a horizontal line representing the limit of ΔSOD = 1.0 so that $B\,\Delta$SOD = 0.75.

Next, consider how the area (expressed in a dimensionless form) depends on x. By eliminating ΔSOD and solving for A we arrive at

$$\frac{A}{P_{max}/(P/A)^0} = 1 + x. \tag{13}$$

However, for $B < 1$, a different line prevails for large L

$$\frac{A}{P_{max}/(P/A)^0} = \frac{x}{B}. \tag{14}$$

The reason for the difference is that ΔSOD cannot exceed 1.0 and, as shown in Fig. 1, this results in a change of slope when $B < 1$ and P_{max}/A goes to zero. Eq. (13) for the dimensionless area can be regarded as limited by power and energy simultaneously. However Eq. (14), which passes through the origin, represents a purely energy-limited battery. Fig. 3 shows the expected dependence of the quantity $A/(P_{max}/(P/A)^0)$ on x. Two straight lines are shown and

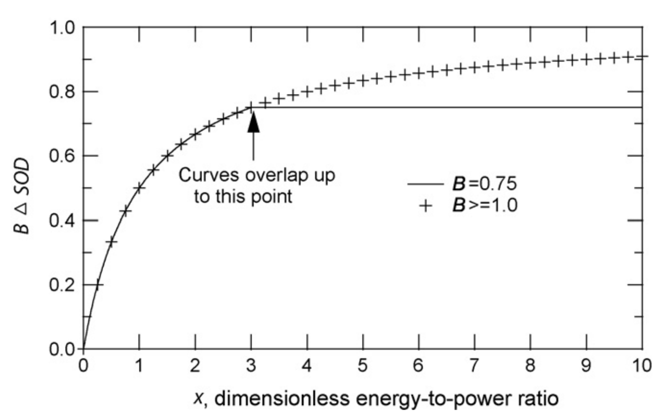

Fig. 2. Capacity usage as a function of the dimensionless energy-to-power ratio. For $B \geq 1.0$, there is a single curve with an asymptote at $B\,\Delta$SOD = 1.0, while for $B < 1.0$, the asymptote for $B\,\Delta$SOD lies below 1.0. The case of $B = 0.75$ is shown.

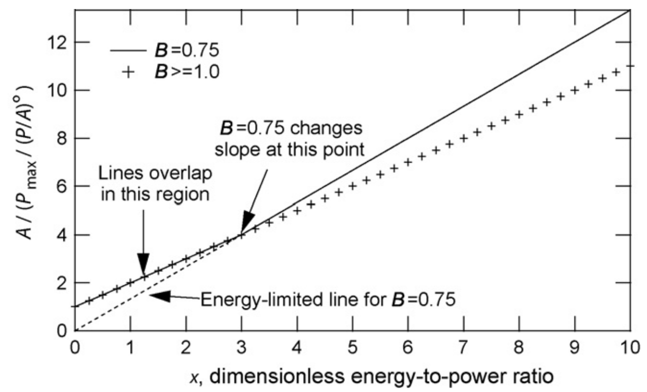

Fig. 3. Dimensionless area as a function of the dimensionless energy-to-power ratio. There is a single line for $B \geq 1.0$, while for $B < 1.0$ there is a change of slope at the point $x = B/(1 - B)$.

the intersection point is at $x = B/(1 - B)$. For $B \geq 1.0$, there is a single line with a non-zero value of $A/(P_{max}/(P/A)^0)$ at $x = 0$ and a slope of 1. For $B < 1.0$, the curve will go through a bend, having a slope of $1/B$ up to the point $x = B/(1 - B)$, and thereafter a slope of 1.

A brief reiteration of the variables and parameters we are using may be helpful. The total separator area, A, is the most important variable for the design of a given battery system for a given vehicle application. Other quantities, such as the parameter B which describes the shape of the pulse-power capability curve, the average cell potential $\langle V\rangle$, the maximum pulse-power capability $(P/A)^0$, and the area-specific capacity Q are all parameters that either depend on the cell chemistry, or can only be changed by altering the cell design (*e.g.*, electrode thicknesses, volume fractions, particle sizes, *etc.*). ΔSOD and the driving distance, L, can be calculated from these quantities using the equations presented above.

4. Applications of the simplified model

In this section we show how to apply the simple model developed in the previous section to common performance representations and practical problems encountered by battery designers. First, consider capacity usage in a HEV. According to USCAR, a minimum power-assist HEV should have a pulse-power capability of 25 kW and an available energy of 300 Wh [5]. To determine the required area for a HEV with these specifications, insert the appropriate values into Eq. (13), and to find the value of ΔSOD use Eq. (11). We use a value of 1625 W m^{-2} for $(P/A)^0$ and a value of 46.2 Wh m^{-2} for $Q\langle V\rangle$. These values are based on the following values characteristic of a lithium-ion cell for a HEV application: $R = 20 \times 10^{-4}$ ohm m^2, $U = \langle V\rangle = 3.8$ V, $V_{min} = 2.5$ V, and $Q = 12.2$ Ah m^{-2} (based on an electrode thickness of 45 μm, a volume fraction of active material of 0.45, an available specific capacity of 150 mAh g^{-1}, and an active material density of 4.0 g cm^{-3}). Table 1 summarizes the results for the separator area and ΔSOD, and Fig. 4 shows the SOD usage superposed on the pulse-power capability curve. From Table 1 we see that a value of B corresponding to a relatively flat pulse-power capability ($B = 0.75$) results in a

Table 1
HEV capacity usage and separator area for three different values of B, a parameter that characterizes the shape of the pulse-power capability curve, with $(P/A)^0 = 1625$ W m^{-2} and $Q\langle V\rangle = 46.2$ Wh m^{-2}

	B		
	0.75	1	1.25
ΔSOD	0.384	0.351	0.322
A (m^2)	21.6	23.7	25.8

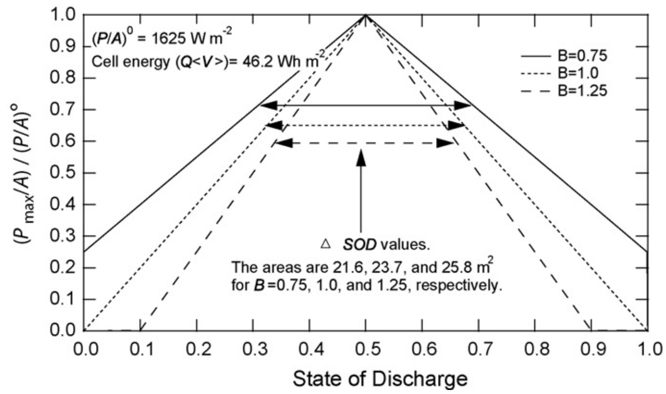

Fig. 4. ΔSOD values superposed on the pulse-power capability curves for the HEV goals given by USCAR (an energy of 300 Wh, and a power of 25 kW over the SOD range that the energy requirement is satisfied). .

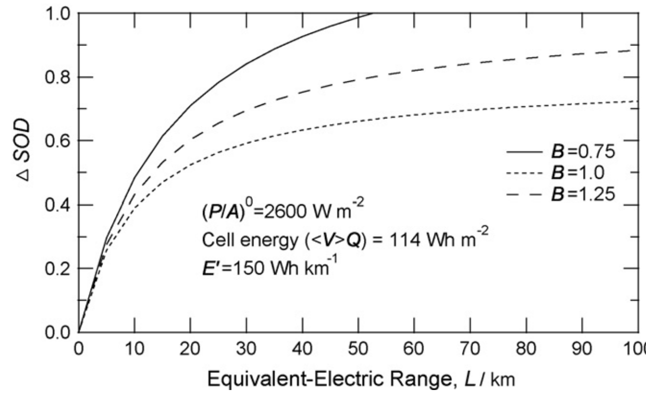

Fig. 6. Capacity usage as a function of the equivalent-electric range for a PHEV. The power requirement is 45 kW, and the energy requirement depends on the equivalent-electric range.

smaller required separator area and a larger SOD range. We stress that even for a HEV, both power and energy goals must be met simultaneously, and a change in either requirement results in a different battery size. In this sense, design of a battery for a HEV is qualitatively similar to the design of a battery for a PHEV, with the difference that a HEV battery requires a high power-to-energy ratio, while a PHEV requires a low power-to-energy ratio. For a PHEV application the lines in Fig. 4 would be drawn at a lower value of $(P_{max}/A)/(P/A)^0$, and the SOD range would be larger.

Second, consider capacity usage in a PHEV. Fig. 5 shows a dimensional presentation of Fig. 3; we use a power requirement of 45 kW, a value of 150 Wh km^{-1} for E', and a value of 2600 W m^{-2} for $(P/A)^0$ and 114 Wh m^{-2} for $Q\langle V\rangle$. The latter two values are based on the following values characteristic of a lithium-ion cell for a PHEV application: $R = 12.5 \times 10^{-4}$ ohm m^2, $U = \langle V\rangle = 3.8$ V, $V_{min} = 2.5$ V, and $Q = 30.0$ Ah m^{-2} (based on an electrode thickness of 100 μm, a volume fraction of active material of 0.5, an available specific capacity of 150 mAh g^{-1}, and an active material density of 4.0 g cm^{-3}). All of the curves have the same intersection with the ordinate, though they each have a different initial slope. The curve for $B = 0.75$ has a change in slope at a range of *ca.* 53 km, and after this point the slope of the line for $B = 0.75$ and $B = 1.0$ are identical. The shift is from the relationship given in Eq. (13) to that given in Eq. (14). The reason for the change in the slope of the $B = 0.75$ line can be clarified through a consideration of Fig. 6. Here we can see that at an equivalent-electric range of *ca.* 53 km, the value of ΔSOD reaches 1.0, which will cause the area to change at a different rate with respect to the equivalent-electric range. The results shown in Figs. 5 and 6 have implications for battery design; they show that a relatively flat pulse-power capability (small values of B) should result in a smaller battery for a given equivalent-electric driving range.

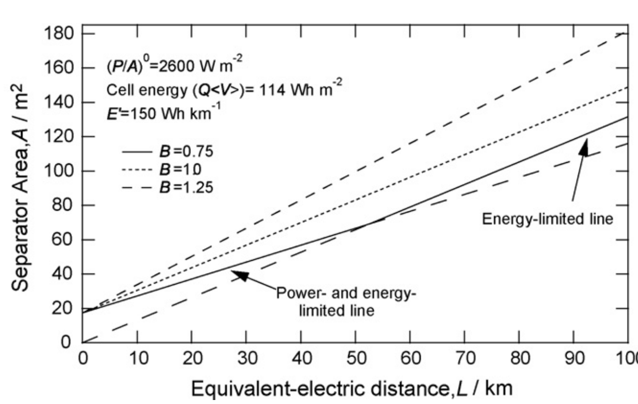

Fig. 5. Separator area as a function of the equivalent-electric range for a PHEV. The power requirement is 45 kW, and the energy requirement depends on the equivalent-electric range.

Finally, consider how the magnitude of the cell equilibrium potential and cell resistance influence the capacity usage and battery size. These are important considerations as developers consider whether it is better to use high-potential systems that typically form resistive films on the electrode surfaces (the anode in particular), or to shift to lower-potential systems without film formation on the electrode surfaces. The value of $(P/A)^0$, which is chemistry- and design-specific, can be approximated by Eqs. (4) or (5) for a value of U and R at SOD = 0.5. Rewriting Eq. (12) for x, and assuming that $\langle V\rangle = U$, we find

$$x = \frac{E'LBc_1U}{P_{max}QR}. \tag{15}$$

Thus, the dimensionless energy-to-power ratio is proportional to the cell potential, U, and inversely proportional to the cell resistance, R. Considering Fig. 2, increasing U while holding R constant would shift the value of $B\,\Delta$SOD higher, all else equal. Note that doubling the value of x at low values of x leads to a larger increase in $B\,\Delta$SOD than a doubling of x at a high value of x. We can rewrite Eq. (10) as

$$\Delta\text{SOD} = \frac{E'L}{E'LB + (RQP_{max}/(c_1U))} \tag{16}$$

This provides a way to determine the change in the SOD usage for a change in U or R, all else equal.

Next, how does the area depend on the value of U and R? Substitute Eq. (16) into Eq. (13):

$$A = \frac{P_{max}}{c_1U^2/R} + \frac{E'LB}{QU}. \tag{17}$$

Here, we see that both the intercept and the slope depend on the value of U. Finally, we can solve for the dependence of the SOD range on the area, which results in

$$B\ \Delta\text{SOD} = 1 - \frac{P_{max}R}{c_1U^2A}. \tag{18}$$

Eq. (18) can be compared with Eq. (6), noting that $(P/A)^0$ has been rewritten according to Eq. (4). Considering Eqs. (17) and (18), we see that while the slope of the area, A, is inversely proportional to the cell potential, ΔSOD is inversely proportional to the square of the cell potential. As an example, in Fig. 7 we plot a battery with $B = 0.75$ (an arbitrary value), $P_{max} = 45$ kW, $c_1 = 0.225$ and two sets of values of R and U. One set uses $R = 20 \times 10^{-4}$ ohm m^2 and

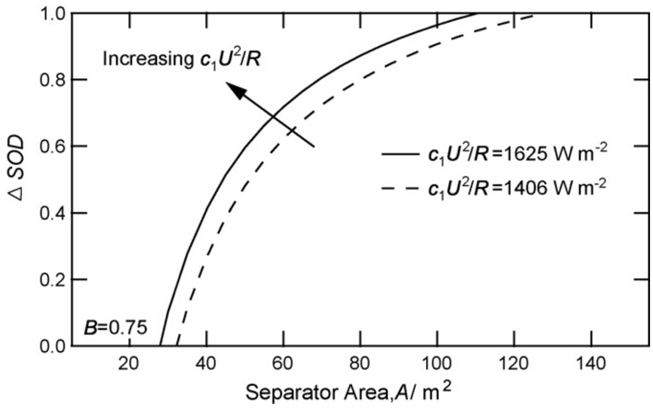

Fig. 7. Capacity usage as a function of separator area for two different hypothetical lithium-ion cell chemistries described in the text. One corresponds to a high-potential, high-resistance system with a Li_xC_6 negative electrode and the other corresponds to a low-resistance, low-potential system with a $Li_{4+3x}Ti_5O_{12}$ negative electrode.

U = 3.8 V so that c_1U^2/R = 1625 W m^{-2} (corresponding to lithium-ion system with a graphite negative electrode), and the other set uses $R = 10 \times 10^{-4}$ ohm m^2 and U = 2.5 V so that c_1U^2/R = 1406 W m^{-2} (corresponding to a lithium-ion system with a $Li_{4+3x}Ti_5O_{12}$, high-potential negative electrode). Here we see that the value of ΔSOD for a given area is higher for the cell with a relatively high value of U and R. Note that the separator area for ΔSOD = 0 corresponds to the minimum separator area required to meet the power requirement for an energy requirement of zero, and in both cases shown here the value of ΔSOD reaches a maximum value of 1 and then levels off. After the value of ΔSOD reaches 1.0, the battery is energy-limited in the sense that it has more than enough power to meet the power requirements. This plot clarifies the importance of the parameter c_1U^2/R for battery design.

5. Conclusions

For a sample set of pulse-power capability curves with a shape governed by the parameter B, a simple model can be used to show a Langmuir-like dependence of capacity usage on a dimensionless energy-to-power ratio, and a linear dependence of a dimensionless area on a dimensionless energy-to-power ratio. The former means that capacity usage increases rapidly at low values of the dimensionless energy-to-power ratio, while it increases slowly at high values of the dimensionless energy-to-power ratio. In dimensional terms, the parameter B (which describes the shape of the pulse-power capability curve), $Q\langle V\rangle$, and $(U - V_{min})V_{min}/R$ are each important values for determining capacity usage and battery size. An ideal cell chemistry would have a small value of B (indicating a relatively flat pulse-power capability), a large value of $Q\langle V\rangle$, and a large value of $(U - V_{min})V_{min}/R$, resulting in a relatively high capacity usage and small battery size. The results in this work are extended to more detailed simulations presented in a companion paper [4]. Finally, while this work has focused on performance, other factors that influence the choice of a battery for a vehicle application include cost, safety, and lifetime.

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